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Excitation of Surface Waves

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EXCITATION OF SURFACE WAVES

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Abstract

It is shown how to locate a dipole source above a dielectric surface so as to either (a) produce as pure a surface wave as possible or (b) maximize the amount of energy carried by the surface wave.

1. Introduction

Since the appearance of Sommerfeld's classical paper[1] on the field produced by a dipole above a plane dielectric earth, a considerable amount of attention has been devoted to the problem of the excitation of surface waves over dielectric surfaces. The two questions which are of practical importance are these: 1) to determine under what conditions a surface wave exists and 2), if it exists, to determine the type of source which is the most efficient generator of the surface wave.

Though it is now generally accepted that the surface wave due to a dipole above a dielectric earth is nowhere predominant on the earth's surface, the question as to whether there exists a source distribution which produces a surface wave predominant in some region on the earth's surface still remains unanswered. To answer this, an investigation of the propagation above a plane earth when the source distribution is arbitrary is necessary. This problem has been considered both by Kay[2] and one of the authors[3] and it has been shown that the surface wave and the radiation field terms may be obtained from a knowledge of the far-field radiation pattern of the original source distribution. From these results it is theoretically possible to combine sources with different far-field patterns in order to determine the most suitable source distribution for generating surface waves. Since this approach requires a large amount of tedious numerical work, it seems desirable to attack the problem in a more analytic manner.

This has been done by Cullen in a recent paper[4]. There he considers the excitation of surface waves by a horizontal slot placed above a plane surface on which an impedance type of boundary condition holds. For a reactive type boundary condition it is known that a surface wave exists. Cullen discusses two different criteria for determining that height of source above the ground plane which is most efficient in exciting the surface wave. By the first criterion the most efficient source is that which produces an almost pure surface wave (this concept will be explained later) near the guiding surface. By the second criterion the most efficient source is the one which produces a field such that the ratio of energy per unit area of wavefront in the surface wave to the sum of energy per unit area of wavefront in the radiated field and the surface wave is a maximum. Cullen obtains values of the optimum source height for each of these criteria. However, when discussing the second criterion, he finds it necessary to use numerical methods and thus obtains the optimum source height only for one particular value of the surface impedance.

The present work was suggested by Cullen's paper but it is more general in that both two- and three-dimensional problems are investigated and a more general type of guiding surface is employed. The surface is assumed to be a slab of dielectric resting on a ground plane on which an impedance-type boundary condition holds. The dielectric need not be homogeneous; in fact, it may consist of layers of different materials, but we assume that the permittivity of the dielectric varies only in the direction perpendicular to the ground plane. When investigating the field produced by a source above this guiding surface, we find it desirable to expand the source into plane waves of all wavelengths in a direction perpendicular to the ground plane and not into plane waves in a direction parallel to the ground plane, as Cullen does. With our type of expansion we find that the only property of the guiding surface that we need is its impedance $Z(k)$ to an incoming normally incident plane wave of wave number k. The optimum height h_1 above the surface at which a slot must be placed to provide a nearly pure surface wave is then determined to be

$$h_1 = \frac{Z_0}{k_0 X(0)}$$

where k_0 is the wave number, Z_0 the characteristic impedance of free space, and where we put

$$Z(k) = R(k) + i\,X(k),$$

so that $X(k)$ is the reactance of the guiding surface. This result generalizes that obtained by Cullen.

We consider also a second criterion for launching efficiency which differs from that proposed by Cullen. The criterion is as follows: That height h_2 of source is most efficient for which the energy in the radiated field per unit area of wavefront is a minimum. If $Z(k)$ is purely reactive we find that h_2 is given by the following formula:

$$h_2 = \frac{\sqrt{2}}{k_0} \cot^{-1} \left\{ \sqrt{2}\, Z\left(\frac{k_0}{\sqrt{2}} \right) \right\} .$$

In the case considered by Cullen these two criteria give practically the same results for h_2 as those he obtained.

The formula for finding the most efficient launching height h_2 may be given the following physical interpretation: Consider a plane wave with wave number k_0 travelling toward the interface in a direction making a 45° angle with the vertical; then h_2 is that height at which the total field produced by this wave and the reflected wave is a minimum.

2. Outline of results

Suppose that the dielectric material extends from $z = -z_0$ to $z = 0$ and that there is free space for $z > 0$. We suppose that the exciting source is a horizontal magnetic slot parallel to the y-axis and passing through the point $z = h$, $x = a$. The field quantities can be expressed in terms of H_y as follows:

$$\underline{H} = (0, H_y, 0)$$
$$\underline{E} = (i\omega\epsilon)^{-1} \left(\frac{\partial}{\partial z}, 0, -\frac{\partial}{\partial x} \right) H_y .$$

There are two methods for obtaining H_y. One, which Cullen uses, is to expand the source in terms of waves travelling along the x-axis. The other, which is used in this paper, is to expand the source in terms of waves which come in and are reflected along the z-axis.

Suppose that a wave e^{-ikz} comes in along the z-axis. Because of the presence of the dielectric, this wave will produce a reflected wave $\rho(k)\, e^{ikz}$ for $z > 0$.

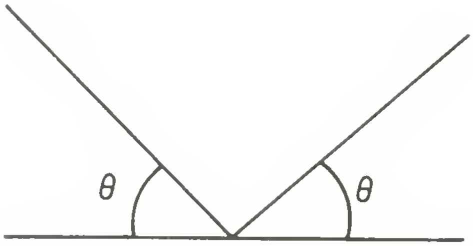
Here k will be any real number from 0 to ∞. However, if the source is oscillating with frequency ω, thus producing waves whose wave number in free space is k_o, then the incoming wave will be

$$e^{-ikz} \exp\left[i \sqrt{k_o^2-k^2}\ |x-a| \right]$$

and the reflected wave will be

$$\rho(k)\ e^{ikz} \exp\left[i \sqrt{k_o^2-k^2}\ |x-a| \right] .$$

Notice that this may be interpreted as a wave coming in at an angle θ to the x-axis and then reflected at the x-axis (see diagram below). Here $\sin\theta = k/k_o$.



When we take into account the fact that the source produces plane waves travelling in all directions we find (see (12')) that H_y will contain a term proportional to

$$\int_{-\infty}^{\infty} \frac{\exp\left[i \sqrt{k_o^2-k^2}\ |x-a| \right] e^{ik(z-h)}}{\sqrt{k_o^2 - k^2}}\ dk$$

$$+ \int_{-\infty}^{\infty} \rho(k)\ \frac{\exp\left[i \sqrt{k_o^2-k^2}\ |x-a| \right] e^{ik(z+h)}}{\sqrt{k_o^2 - k^2}}\ dk .$$

The first integral is just the field that would be produced by the source in free space if the dielectric were not present, and the second integral is the reflected field that is produced by the dielectric. The sum of these two integrals will give the radiation field.

Let us consider $\rho(k)$. There may be a complex value of k with positive imaginary part, call it k_1, such that $\rho(k_1)$ is infinite. This value of k_1 will produce a surface wave and we shall have in H_y (see (12)) a term proportional to

$$\exp\left[i\sqrt{k_o^2-k_1^2}\,|x-a|\right] e^{ik_1 z}\, e^{ik_1 h}, \qquad z > 0.$$

Notice that, because the imaginary part of k_1 is positive, this expression represents a wave which travels along the x-axis at a speed faster than light and is exponentially attenuated in the z-direction. This behavior is typical of surface waves.

The value of the reflection coefficient can be found from the impedance of the dielectric to an incoming wave e^{-ikx}. We have, from (17),

$$\rho(k_o\sin\theta) = \frac{\sin\theta + \dfrac{Z(k_o\sin\theta)}{Z_o}}{\sin\theta - \dfrac{Z(k_o\sin\theta)}{Z_o}} .$$

From (14) the radiation field at large distances R from the source and in the direction θ to the x-axis is proportional to

$$\frac{e^{ik_oR}}{\sqrt{k_1R}}\left[e^{-ik_oksin\theta} + \rho(k_o\sin\theta)e^{ik_oksin\theta}\right]$$

Using this we find (see (25)) that the radiated power per unit area of wavefront is proportional to

$$(*) \qquad \int_0^{\frac{\pi}{2}} \left| 1 + \rho(k_o\sin\theta)\, e^{2ik_oksin\theta}\right|^2 d\theta.$$

This formula may be given a simple physical interpretation. The field produced by the source may be decomposed into plane waves travelling in all directions. The plane waves which travel in the direction $-\theta$ will be reflected back to the height h of the source with a reflection factor $\rho(k_o\sin\theta)e^{2ik_oksin\theta}$. The total field at a distance R will then be the sum of the direct plane wave whose amplitude is A, say, and the reflected plane wave with amplitude $A\rho(k_o\sin\theta)e^{2ik_oksin\theta}$. The total energy power in the wavefront will be the integral of the absolute square of the value of the field, and in this way we get the equation (*).

Put $\rho = |\rho|\, e^{+2i\gamma}$, where $|\rho|$ and γ are real. Using this, we find that the radiation pattern $F(\theta)$ is given by

$$F(\theta) = \left| e^{-ik_0 h \sin\theta} + |\rho|\, e^{i(k_0 h \sin\theta + 2\gamma)} \right|$$

$$= \left[1 + |\rho|^2 + 2|\rho| \cos 2(k_0 h \sin\theta + \gamma) \right]^{1/2} .$$

Note that the value of the reflection coefficient can be obtained from the impedance of the dielectric to an incoming wave e^{-ikx}. We have, from (17),

$$\rho(k_0 \sin\theta) = \frac{\sin\theta + \dfrac{Z(k_0 \sin\theta)}{Z_0}}{\sin\theta - \dfrac{Z(k_0 \sin\theta)}{Z_0}} .$$

In case Z is a pure reactance, this formula shows that $|\rho| = 1$, and a knowledge of the far-field pattern $F(\theta)$ enables us to obtain γ and thus also $\rho(k_0 \sin\theta)$. Theoretically, this information about ρ permits us to obtain the values of θ for which $\rho(k_0 \sin\theta) = \infty$, that is, for which surface waves exist. This agrees with the results in [2] and [3]. However, in practice it is very difficult to find the surface wave this way because the far-field pattern gives the values of $\rho(k_0 \sin\theta)$ for real angles θ whereas a surface wave corresponds to complex angles θ.

A glance at the Figures will show that, at least in the case where Z is a constant reactance, the radiated power per unit area of wavefront is approximately proportional to $F(\theta)^2$ evaluated at $\theta = \pi/4$. Using this approximation, we may determine the height h_2 for which the radiated power per unit area of wavefront is a minimum. We find

$$k_0\, h_2 \sin\frac{\pi}{4} + \gamma = \frac{\pi}{2} ,$$

or

$$h_2 = \frac{\sqrt{2}}{k_0} \left(\frac{\pi}{2} - \gamma\right).$$

Since in the case of a pure reactance, $\gamma = \tan^{-1} \dfrac{X}{Z_0 \sin \frac{\pi}{4}}$, we get (see (28))

$$h_2 = \frac{\sqrt{2}}{k_0} \cot^{-1}\left(\sqrt{2}\; \frac{X}{Z_0} \right).$$

Going back to the physical interpretation of (*), we can interpret h_2 as follows: The height of source h_2 at which the radiated power per unit area of wavefront is a minimum is approximately the height at which there is complete interference, i.e., zero amplitude between the direct and reflected waves travelling in the 45°-direction to the interface.

For experimental purposes it is useful to be able to produce a field which is as pure a surface wave as possible. Since the surface wave attenuates exponentially in a direction perpendicular to the interface, we consider the field at a point on the interface at a distance x from the source. From (12) the H_y component produced by the surface wave is

$$\frac{1}{2i} \; \frac{e^{i\sqrt{k_0^2-\lambda_1}\, x}}{\sqrt{k_0^2 - \lambda_1}} \quad u(0,\lambda_1)u(h,\lambda_1),$$

whereas from (18) the H_y component produced by the radiation field is

$$\sqrt{\frac{2}{\pi}} \; \frac{Z_0}{Z(0)} \; \frac{\exp\left[i(k_0|x| - \frac{\pi}{4})\right]}{k_0|x|^{3/2}} \quad u_1(0)u_1(h),$$

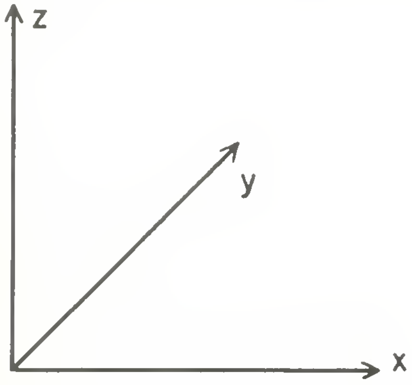
plus terms of order $|x|^{-5/2}$. By proper choice of the launching height, the above term in the radiation field of order $|x|^{-3/2}$ can be made zero, and therefore the total field, which is the sum of the surface wave and the radiation field, becomes, at least at large distances from the source, very nearly a pure surface wave. The height at which this term in the radiation field vanishes is given by (19) as

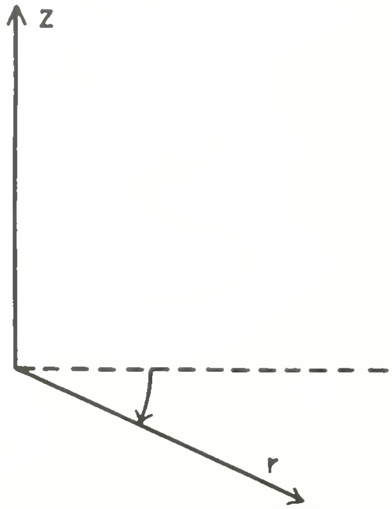
$$h = \frac{iZ_0}{k_0 Z(0)} .$$

3. Mathematical formulation

Consider a ground plane at $z = -z_o$ and suppose that the permittivity of the medium above it depends on the z-coordinate only. For example, the medium might be a dielectric slab from $z = -z_o$ to $z = 0$ and then free space from $z = 0$ to $z = \infty$; or the medium might be composed of layers of dielectric of arbitrary thickness and permittivity from $z = -z_o$ to $z = 0$ and free space above; or the permittivity of the medium might vary continuously with height from $z = -z_o$ to $z = 0$ and again have free space above. In all cases we shall assume that the medium between $z = 0$ and $z = \infty$ is free space and that it contains a source of electromagnetic radiation. Finally, we assume that the electromagnetic field satisfies an impedance-type boundary condition at the ground plane $z = -z_o$.

We shall treat both a two- and a three-dimensional problem simultaneously with the help of the following coordinate systems:





In the first diagram above, the source of the electromagnetic field is a magnetic line dipole placed at $z = h$, $x = a$ and extended parallel to the y-axis. Because of the geometry, the field produced will be independent of y. We shall call this the two-dimensional problem and denote it by Case I.

In the second diagram above, the source is a circular magnetic dipole located at $z = h$, $r = a$. Because of the geometry, the field produced will be independent of θ. We shall call this the three-dimensional problem and denote it by Case II. In both

The simple-harmonic form for Maxwell's equations is

$$\text{curl } \underline{H} = -i\,\omega\,\varepsilon\,\underline{E}$$

$$\text{curl } \underline{E} = i\,\omega\,\mu\,\underline{H}$$

where $\underline{E}$, $\underline{H}$ are the electric and magnetic field intensities and ε and μ are the permittivity and magnetic permeability. Note that by assumption ε is a function of z and μ is a constant.

In Case I, because the source is a magnetic line dipole, the field is completely determined by the value of H_y. We have

$$H_x = E_y = H_z = 0$$

and

$$E_x = (-i\omega\varepsilon)^{-1}\,\frac{\partial H_y}{\partial z}\,, \qquad E_z = (-i\omega\varepsilon)^{-1}\,\frac{\partial H_y}{\partial z}\,.$$

The field component H_y satisfies the equation

$$(1) \qquad \frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial z^2} + \varepsilon\,\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\,\frac{\partial H_y}{\partial z}\right) + \omega^2 \varepsilon\mu H_y = \delta(z-h)\delta(x-a)\,.$$

Put $H_y = \varepsilon^{1/2}\,H_y'$ and (1) becomes

$$(2) \qquad \frac{\partial^2 H_y'}{\partial x^2} + \frac{\partial^2 H_y'}{\partial z^2} + \omega^2 \varepsilon'\mu\, H_y = \frac{1}{\sqrt{\varepsilon}}\,\delta(z-h)\delta(x-a)$$

where

$$(3) \qquad \omega^2 \varepsilon'\mu = \omega^2 \varepsilon\mu + \frac{1}{2\varepsilon}\left\{\frac{d^2\varepsilon}{dz^2} - \frac{3}{2}\left(\frac{d\varepsilon}{dz}\right)^2\right\}.$$

In Case II, because of our assumption about the source, the field will be completely determined by the value of H_θ. We have

$$H_z = H_r = E_\theta = 0$$

and

$$E_r = (i\omega\varepsilon)^{-1} \frac{\partial H_\theta}{\partial z} , \qquad E_z = (-i\omega\varepsilon)^{-1} r^{-1} \frac{\partial}{\partial r} (rH_\theta).$$

The field component H_θ will satisfy the equation

$$(4) \qquad \frac{\partial^2 H_\theta}{\partial z^2} + \frac{\partial^2 H_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial H_\theta}{\partial r} + \left(\omega^2 \varepsilon\mu - \frac{1}{r^2}\right) H_\theta + \varepsilon \frac{\partial}{\partial z} \left(\frac{1}{\varepsilon} \frac{\partial H_\theta}{\partial z}\right) = \frac{\delta(z-h)\delta(r-a)}{2\pi r} .$$

Put $H_\theta = \varepsilon^{1/2} H_\theta'$ and the following equation is obtained:

$$(5) \qquad \frac{\partial^2 H_\theta'}{\partial z^2} + \frac{\partial^2 H_\theta'}{\partial r^2} + \frac{1}{r} \frac{\partial H_\theta'}{\partial r} + \left(\omega^2 \varepsilon' \mu - \frac{1}{r^2}\right) H_\theta = \frac{\delta(z-h)\delta(r-a)}{2\pi r \ \varepsilon^{1/2}}$$

where ε' is defined by (3).

We set

$$\omega^2 \varepsilon' \mu = \omega^2 \varepsilon_o \mu - q(z) = k_o^2 - q(z)$$

where ε_o is the permittivity in free space. Note that by our assumption $q(z) = 0$ if $z > 0$. If we denote the operator $q(z) - \frac{\partial^2}{\partial z^2}$ by L, the formal solution of equation (2) is

$$(6) \qquad H_y' = \varepsilon^{-1/2} \ \frac{\exp\left[i \sqrt{k_o^2 - L} \ |x-a|\right]}{2i \sqrt{k_o^2 - L}} \ \delta(z-h)$$

and the corresponding formal solution of (5) is

$$(7) \qquad H_\theta' = \frac{\varepsilon^{-1/2}}{4i} \ J_1\left(\sqrt{k_o^2 - L} \ a\right) \ H_1^{(1)}\left(\sqrt{k_o^2 - L} \ r\right) \ \delta(z-h), \qquad r > a.$$

These results will be interpreted in the following section.

4. Interpretation of solution

To interpret (6) and (7) is is necessary to express $\delta(z-h)$ in terms of the eigenfunctions of the operator L.

Consider the eigenvalue problem

$$(8) \qquad Lu(z) = -\frac{d^2u}{dz^2} + q(z)u = \lambda u$$

where $u(z)$ satisfies the given impedance-type boundary condition at $z = -z_o$. If $q(z)$ and the impedance-type boundary condition at $z = -z_o$ are real, there may exist negative values of λ such that (8) has a non-zero solution satisfying the given boundary condition at $z = -z_o$. If $q(z)$ (or the impedance-type boundary condition) is not real, there may exist complex values of λ with negative real parts such that (8) has a non-zero solution satisfying the given boundary conditionat $z = -z_o$. In either case we shall denote these values of λ by λ_j and the corresponding normalized solution of (8) by $u(z,\lambda_j)$. Note that $u(z,\lambda_j)$ vanishes exponentially as $z \to \infty$.

Every positive value of λ will be in the continuous spectrum of the operator L. Put $\lambda = \kappa^2$ and denote by $u(z,\kappa)$ that solution of (8) which satisfies the boundary condition at $z = -z_o$ and which as $z \to \infty$ has the asymptotic form

$$(9) \qquad u(z,k) \sim e^{-ikz} + \rho(\kappa)e^{ikz} .$$

Here $\rho(\kappa)$ is the reflection coefficient for the incoming plane wave e^{-ikz} incident on the dielectric at $z = 0$. Since $q(z) = 0$ for $z > 0$, formula (9) is an equality for $z > 0$. We have[5]

$$(10) \qquad \delta(z-h) = \sum_j u(z,\lambda_j)u(h,\lambda_j) + \frac{1}{2\pi}\int_0^\infty u(z,k)\,\overline{u(h,k)}\,dk .$$

Here $\overline{u(z,k)}$ is that solution of (8) satisfying the boundary condition at $z = -z_o$ which for $z > 0$ has the form

$$\overline{u(z,k)} = e^{ikz} + \rho(-k)e^{-ikz} \quad . \tag{11}$$

If $q(z)$ and the impedance-type boundary condition at $z = -z_o$ are real, the value of $\rho(-k)$ is the complex conjugate of the value of $\rho(k)$ and hence $\overline{u(z,k)}$ is the complex conjugate of $u(z,k)$. However, in all cases, comparison of (9) and (11) shows that $\rho(-k) = \rho(k)^{-1}$.

If the expression (10) is substituted in equations (6) and (7) and if we use the fact that u satisfies (8), we get the following:

$$\begin{aligned} H_y' = {} & \frac{\varepsilon^{-1/2}}{2i} \sum_j \frac{\exp\left[i\sqrt{k_o^2-\lambda_j}\,|x-a|\right]}{\sqrt{k_o^2-\lambda_j}}\, u(z,\lambda_j)u(h,\lambda_j) \\ & + \frac{\varepsilon^{-1/2}}{4\pi i} \int_0^\infty \frac{\exp\left[i\sqrt{k_o^2-k^2}\,|x-a|\right]}{\sqrt{k_o^2-k^2}}\, u(z,k)\,\overline{u(h,k)}\,dk\,, \end{aligned} \tag{12}$$

$$\begin{aligned} H_\theta' = {} & \frac{\varepsilon^{-1/2}}{4i} \sum_j J_1\left(\sqrt{k_o^2-\lambda_j}\;a\right) H_1^{(1)}\left(\sqrt{k_o^2-\lambda_j}\;r\right) u(z,\lambda_j)u(h,\lambda_j) \\ & + \frac{\varepsilon^{-1/2}}{8\pi i} \int_0^\infty J_1\left(\sqrt{k_o^2-k^2}\;a\right) H_1^{(1)}\left(\sqrt{k_o^2-k^2}\;r\right) u(z,k)\,\overline{u(h,k)}\,dk\,. \end{aligned} \tag{13}$$

The first term on the right-hand side in equation (12) or equation (13) is the surface wave, whereas the integral term in these equations is the radiation field. These formulas show that the existence of a surface wave depends on the existence of eigenvalues of (8) with negative real parts. Thus existence depends only on the value of $q(z)$ and the impedance-type boundary condition at the ground plane, that is, only on the electromagnetic properties of the medium in the z-direction and not on the number of dimensions.

5. Asymptotic behavior of the field

We shall obtain the asymptotic behavior of the integrals in (12) and (13) and from these obtain the asymptotic value of the electromagnetic field components.

Using (9) and (11), we see that

$$u(z,k)\overline{u(h,k)} = e^{-ik(z-h)} + e^{ik(z+h)} + \rho(k)e^{ik(z+h)} + \rho(-k)e^{-ik(z+h)} .$$

Putting this expression into (12) and (13), we find that the integral term becomes

$$\int_{-\infty}^{\infty} \frac{\exp\left[i\sqrt{k_o^2-k^2}\;|x-a|\right]}{\sqrt{k_o^2 - k^2}} \left[e^{+ik(z-h)} + \rho(k)e^{ik(z+h)}\right] dk \tag{12'}$$

in Case I and

$$\int_{-\infty}^{\infty} J_1\left(\sqrt{k_o^2-k^2}\;a\right) H_1^{(1)}\left(\sqrt{k_o^2-k^2}\;r\right) \left[e^{+ik(z-h)} + \rho(k)e^{ik(z+h)}\right] dk \tag{13'}$$

in Case II.

The method of stationary phase will be used to obtain asymptotic expressions for these integrals. In Case I we put

$$x - a = R\cos\theta, \qquad z = R\sin\theta$$

but in Case II we put

$$r = R\cos\theta, \qquad z = R\sin\theta.$$

For large values of R, the integral in (12) is asymptotically

$$\frac{\varepsilon^{-1/2}}{\sqrt{8\pi}}\;\frac{e^{i(k_oR-\frac{\pi}{4})}}{\sqrt{k_oR}}\left[e^{-ik_oh\sin\theta} + \rho(k_o\sin\theta)e^{+ik_oh\sin\theta}\right] \tag{14}$$

and the integral in (13) is asymptotically

$$(15) \qquad \frac{\varepsilon^{-1/2}}{4\pi} \; \frac{e^{i(k_oR-\frac{3\pi}{4})}}{k_oR} J_1(k_oa\cos\theta)\left[e^{-ik_oh\sin\theta} + \rho(k_o\sin\theta)e^{+ik_oh\sin\theta}\right] .$$

The expressions (14) and (15) are essentially the radiation field parts of H_y and H_θ, respectively. We now consider the value of the radiation field on the surface of the slab, at large distances from the source, that is, at $z = 0$ but $R \gg 0$. In this case θ approaches zero and we can show that the expression in brackets in (14) and (15) approaches zero. This indicates that at $z = 0$ the asymptotic expression for the radiation field is no longer given by (14) and (15).

We shall obtain the correct asymptotic formula for the radiation field on the surface of the slab for R large by evaluating the integrals in (12) and (13) in a different manner. However, first we evaluate $\rho(k_o\sin\theta)$.

At the interface $z = 0$, between the dielectric slab and free space, the ratio E_x/H_y or E_r/H_θ must be continuous. Denote this ratio by $Z(k)$; then

$$(i\omega\varepsilon)^{-1} \frac{\partial H_y}{\partial z} \Big/ H_y = Z(k),$$

or

$$(16) \qquad \frac{1}{u}\frac{du}{dz}\bigg|_{z=0} = i\omega\varepsilon Z(k) = \frac{ik_o}{Z_o} Z(k),$$

where Z_o is the characteristic impedance of free space. Note that $Z(k)$ is the impedance at the interface to an incoming wave e^{-ikz}. For convenience, we shall sometimes write this impedance as Z. For $z > 0$ we have

$$u(z,k) = e^{-ikz} + \rho(k)\, e^{ikz} ;$$

then by (16)

$$\frac{1-\rho}{1+\rho} = -\frac{k_o Z}{k Z_o} ,$$

or

$$\rho = \frac{\dfrac{k}{k_o} + \dfrac{Z}{Z_o}}{\dfrac{k}{k_o} - \dfrac{Z}{Z_o}} .$$

Finally,

$$\rho(k_o \sin\theta) = \frac{\sin\theta + \dfrac{Z(k_o \sin\theta)}{Z_o}}{\sin\theta - \dfrac{Z(k_o \sin\theta)}{Z_o}} . \tag{17}$$

From this formula it is clear that if $Z(0) \neq 0$, the value of $\rho(k_o \sin\theta)$ approaches -1 as θ approaches zero and therefore the bracketed expression in (14) and (15) approaches zero. Also, from (17),

$$\rho(k_o \sin\theta) = -1 - \frac{2Z_o}{Z(0)} \sin\theta$$

for small values of θ. Then

$$u(z,k_o \sin\theta) = e^{-ik_o z \sin\theta} + \rho(k_o \sin\theta) e^{ik_o z \sin\theta}$$

$$= -\frac{2\sin\theta}{k_o}\left[\frac{Z_o}{Z(0)} + ik_o z\right] .$$

We shall denote the bracketed expression in this formula by $u_1(z)$.

Let us return to the integrals in (12) and (13) and consider their asymptotic behavior for large values of x or r. When we make the change of variables

$$k^2 = k_o^2 - \beta^2$$

we have

$$\frac{dk}{\sqrt{k_o^2-k^2}} = \frac{d\beta}{\sqrt{k_o^2-\beta^2}} ;$$

therefore the resulting integrand becomes infinite at $\beta = k_0$, that is, at $k = 0$. This indicates that the asymptotic behavior of the integrals in (12) and (13) can be obtained by expanding the integrand in powers of k. In this way we find that the integral in Case I is

$$(18) \qquad \sqrt{\frac{2}{\pi}} \; \frac{Z_0}{Z(0)} \; \frac{e^{i(k_0|x-a| - \frac{\pi}{4})}}{k_0|x-a|^{3/2}} \, u_1(z) \, u_1(h)$$

and in Case II it is

$$\frac{1}{4\pi} \; J_1 \frac{(k_0 a)}{k_0^2} \; \frac{Z_0}{Z(0)} \; \frac{e^{i(k_0 r - \frac{3\pi}{4})}}{r^2} \, u_1(z) \, u_1(h).$$

If we choose $u_1(h)$ such that $u_1(h) = 0$, then clearly both expressions for the radiation field will vanish. This means that to obtain as pure a surface wave as possible on the interface, we must require the height h of the source to be such that

$$(19) \qquad k_0 h = \frac{iZ_0}{Z(0)} .$$

Since the source is always above the interface, that is, $h > 0$, formula (19) shows that a surface wave will exist only if Z is a positive reactance. In case Z is complex, formula (19) gives a complex value for h, which means that at no real height will $u_1(h) = 0$. However, if we put $Z = R + iX$, a surface wave can exist only if $X > 0$ and we see that $|u_1(h)|$ will be a minimum at that height h for which

$$(20) \qquad k_0 h = + \frac{Z_0}{X(0)} .$$

6. Comparison of power

In this section we compare the radiated power to the power in the surface wave. For simplicity we shall restrict ourselves to Case I. Similar results can be obtained for Case II.

From (12) the amplitude of one surface wave term in H_y is

$$(21) \qquad (2i)^{-1} \frac{e^{i\sqrt{\kappa_o^2-\lambda_1}|x-a|}}{\sqrt{\kappa_o^2-\lambda_1}} \; u(z,\lambda_1)\, u(h,\lambda_1).$$

Note that

$$(22) \qquad u(z,\lambda_1) = (2i\sqrt{\lambda_1})^{1/2}\, e^{i\sqrt{\lambda_1}\, z}, \qquad z > 0.$$

The x-component of the average power is

$$(23) \qquad \frac{1}{2}\,\mathrm{Re}(i\omega\varepsilon)^{-1}\int_0^{\infty} \frac{\partial H_y}{\partial x}\, H_y^{*}\, dz$$

where H_y^* is the complex conjugate of H_y.

Since power orthogonality between surface wave and radiated field occurs only in case λ_1 is real, we shall assume henceforth that λ_1 is real. Then, applying (23) to (21), we find that the average power in the surface wave is

$$(24) \qquad \frac{\beta}{8\sqrt{\kappa_o^2+\beta^2}}\, e^{-2\beta h}$$

if $\lambda_1 = -\beta^2$.

To find the radiated power per unit area of wavefront we must evaluate

$$\frac{1}{2}\,\mathrm{Re}\int_0^{\pi/2} H_y^{*}\, E_\theta\, d\theta$$

or, since

$$E_\theta = (i\omega\varepsilon)^{-1}\frac{\partial H_y}{\partial r},$$

we must evaluate

$$\frac{1}{2}\ \mathrm{Re}(i\omega\varepsilon)^{-1}\int_{o}^{\pi/2}\frac{\partial H_y}{\partial r}\,H_y^{*}\,d\theta.$$

Using (14) multiplied by $\varepsilon^{1/2}$ for the amplitude of the radiated field in H_y, we find that the radiated power per unit width of wavefront is

$$\begin{aligned}&\frac{(\omega\varepsilon)^{-1}}{16\pi k_o}\int_{o}^{\pi/2}\left[e^{-ik_oh\sin\theta}+\rho(k_o\sin\theta)e^{ik_oh\sin\theta}\right]\\&\qquad\left[e^{ik_oh\sin\theta}+\rho(-k_o\sin\theta)e^{-ik_oh\sin\theta}\right]d\theta.\end{aligned}\tag{25}$$

If we assume Z is purely reactive and constant for all κ, we shall get Cullen's results. Put $Z = i\alpha Z_o$; then from (17)

$$\rho(\kappa_o\sin\theta) = \frac{i\ \sin\theta-\alpha}{i\ \sin\theta+\alpha}$$

and (25) becomes

$$(16\pi\omega\varepsilon\kappa_o)^{-1}\ I(h,h)$$

where

$$\begin{aligned}I(h,h) = \int_{o}^{\pi/2}&\left[e^{-ik_oh\sin\theta}-\frac{\alpha-i\ \sin\theta}{\alpha+i\ \sin\theta}\,e^{ik_oh\sin\theta}\right]\\&\left[e^{ik_oh\sin\theta}-\frac{\alpha+i\ \sin\theta}{\alpha-i\ \sin\theta}\,e^{-ik_oh\sin\theta}\right]d\theta.\end{aligned}\tag{26}$$

It is easy to show that

$$I(h,h') = 2\int_0^{\pi/2}\left[\cos\left\{k_o(h-h')\sin\theta\right\} + 2\mathrm{Re}\,\frac{i\sin\theta+\alpha}{i\sin\theta-\alpha}\,e^{-ik_o\sin\theta(h+h')}\right]d\theta$$

$$= \pi\,J_o\left[k_o(h-h')\right] + \pi\,J_o\left[k_o(h+h')\right] - \pi\int_0^{\infty} 2k_o\alpha e^{-k_o\alpha t}\,J_o\left[k_o\left\{t-(h+h')\right\}\right]dt$$

(27)

$$= \pi\left\{J_o\left[k_o(h-h')\right] + J_o\left[k_o(h+h')\right] - \frac{2\alpha e^{-2k_o\alpha k}}{\sqrt{1+\alpha^2}}\right.$$

$$\left. - 2\alpha e^{-2k_o\alpha h}\int_0^{2k_o\alpha h}\rho^t\,J_o\left(\frac{t}{\alpha}\right)dt\right\}.$$

The last integral is fairly easy to compute numerically for various values of α and $\beta = 2k_o\alpha h$. It was found that the values of $I(h,h)$ so obtained are very closely approximated by the $\pi/2$ times the value of the integrand at $\pi/4$.

In Figures 1, 2, 3, 4 we have graphed for various values of α the functions

$$F(\beta) = 1 + J_o\left(\frac{\beta}{\alpha}\right) - \frac{2\alpha e^{-\beta}}{\sqrt{1+\alpha^2}} - 2e^{-\beta}\int_0^{\beta} e^t J_o\left(\frac{t}{\alpha}\right)d\alpha$$

and

$$g(\beta) = \frac{1}{\pi}\left[\frac{2(1-2\alpha^2)}{1+2\alpha^2}\cos\left(\frac{\beta}{\alpha\sqrt{2}}\right) - \frac{4\alpha\sqrt{2}}{1+2\alpha^2}\sin\left(\frac{\beta}{\alpha\sqrt{2}}\right)\right].$$

Here

$$\beta = 2k_o\alpha h, \qquad F(\beta) = \frac{1}{\pi}\,I(h,h)$$

and $2+\pi g(\beta)$ is the value of the integrand in (26) for $\theta = \pi/4$. By our approximation, we should have

$$F(\beta) = 1 + \frac{\pi}{2}\,g(\beta).$$

The graphs show that this relation is satisfied very closely.

When this approximation is used in (25), the radiated power per unit area of wavefront becomes

$$(16\pi k_o \omega\epsilon)^{-1}\left[2 + 2\cos 2\left\{k_o \sin h + \gamma\right\}\right]$$

where

$$\tan\gamma = \frac{\alpha}{\sin \pi/4} = \sqrt{2}\,\alpha.$$

Note that γ is the phase change on reflection from the interface. From this formula we see that the radiated power will be a minimum if

$$k_o \sin h + \gamma = \pi/2$$

i.e., if

$$k_o h = \sqrt{2}\,\cot^{-1}\sqrt{2}\,\alpha. \tag{28}$$

For $\alpha = 1/2$, this formula gives $k_o h = 1.35$, which agrees fairly well with the value 1.26 found by Cullen. The following table compares the values for $k_o h$ found by using (28) with those found by numerical integration of (27).

Table

α	0.2	0.4	0.6	0.8	1.0
$k_o h$ from (27)	1.62	1.37	1.13	.94	0.82
$k_o h$ from (28)	1.83	1.47	1.23	1.02	0.88

We conclude that for values of $\alpha \geq 0.4$, the value h of the most efficient launching height as computed from the approximate formula (28) is within 10% of the height as computed exactly from (27).

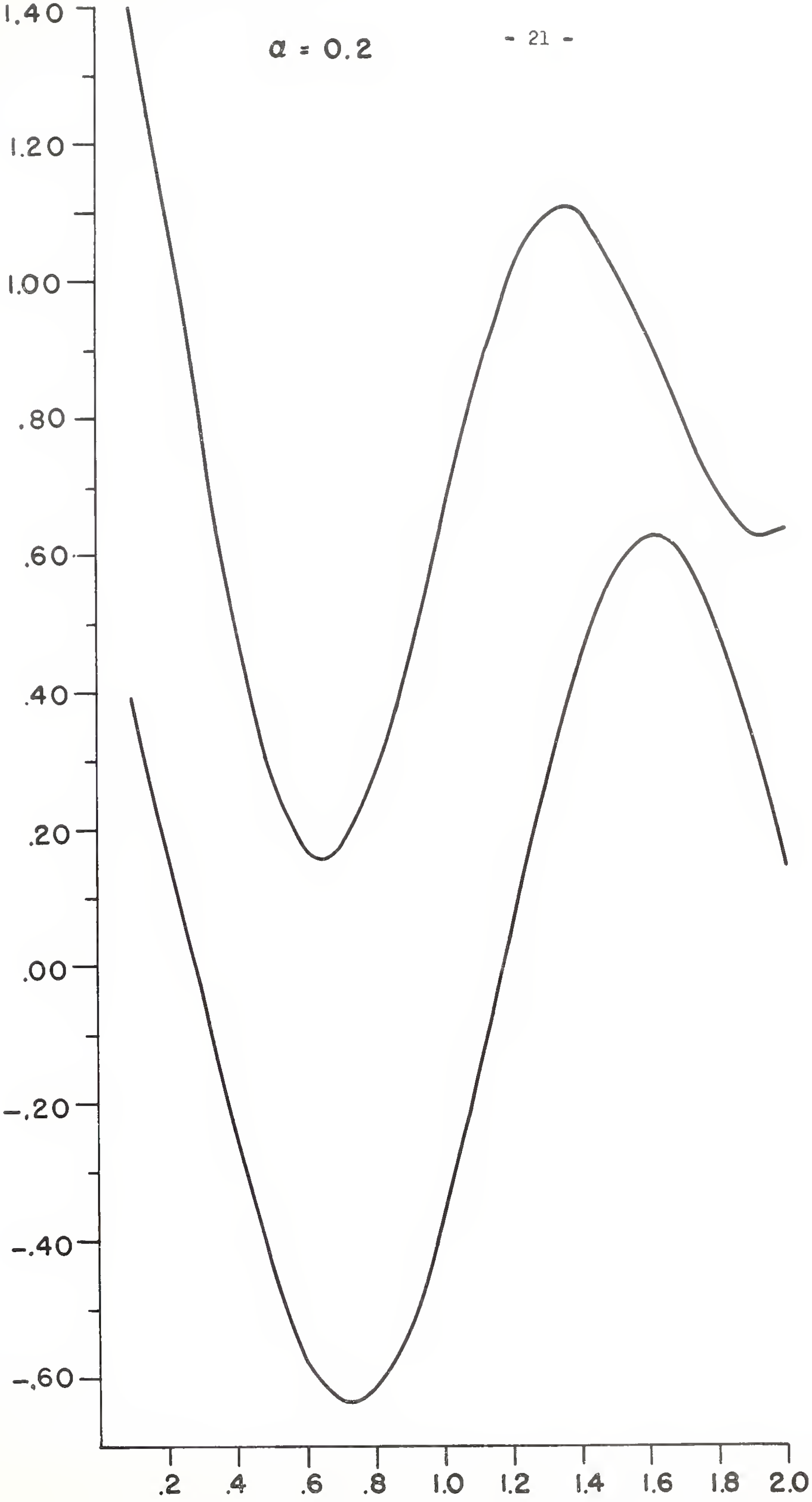


Fig. 1

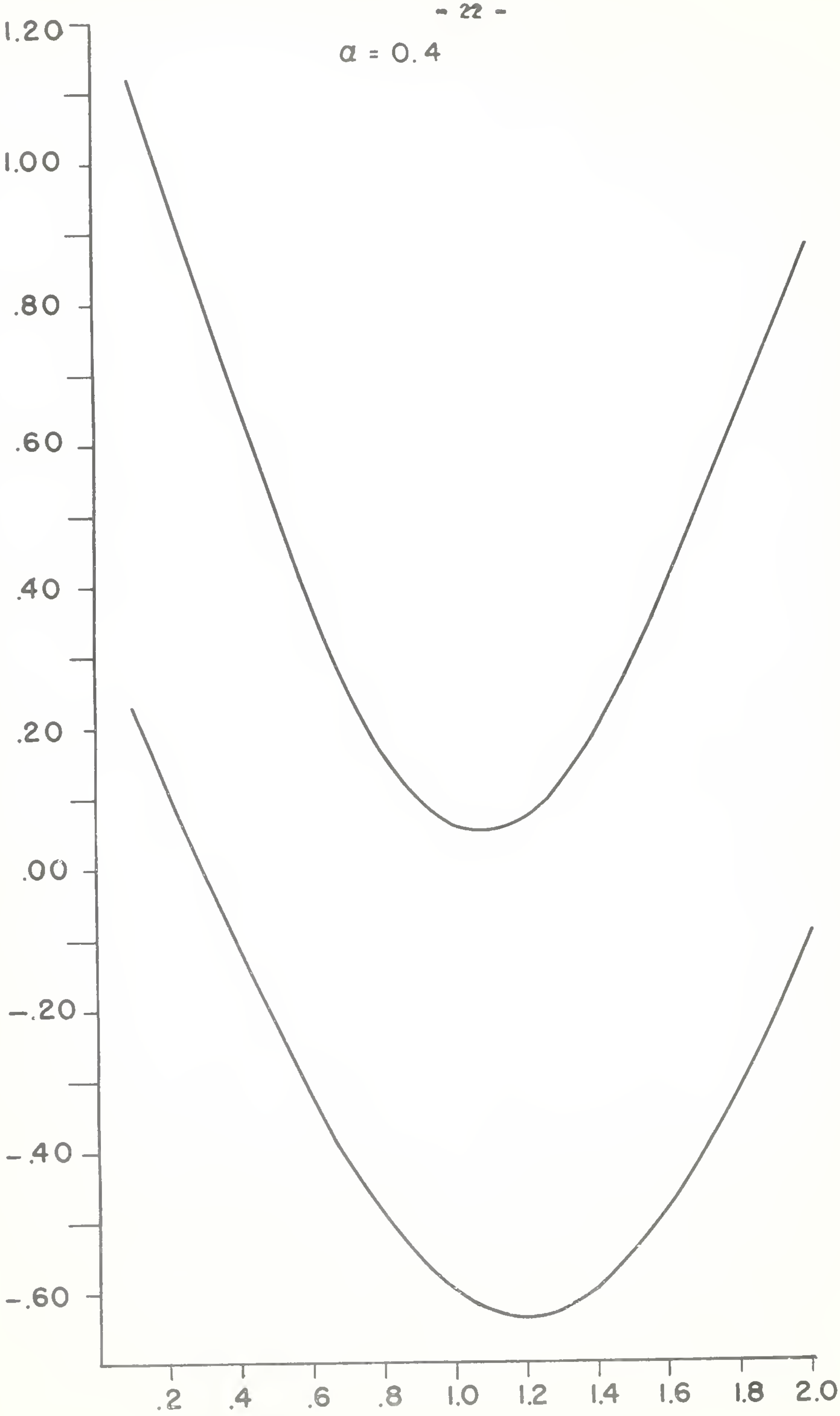


Fig. 2

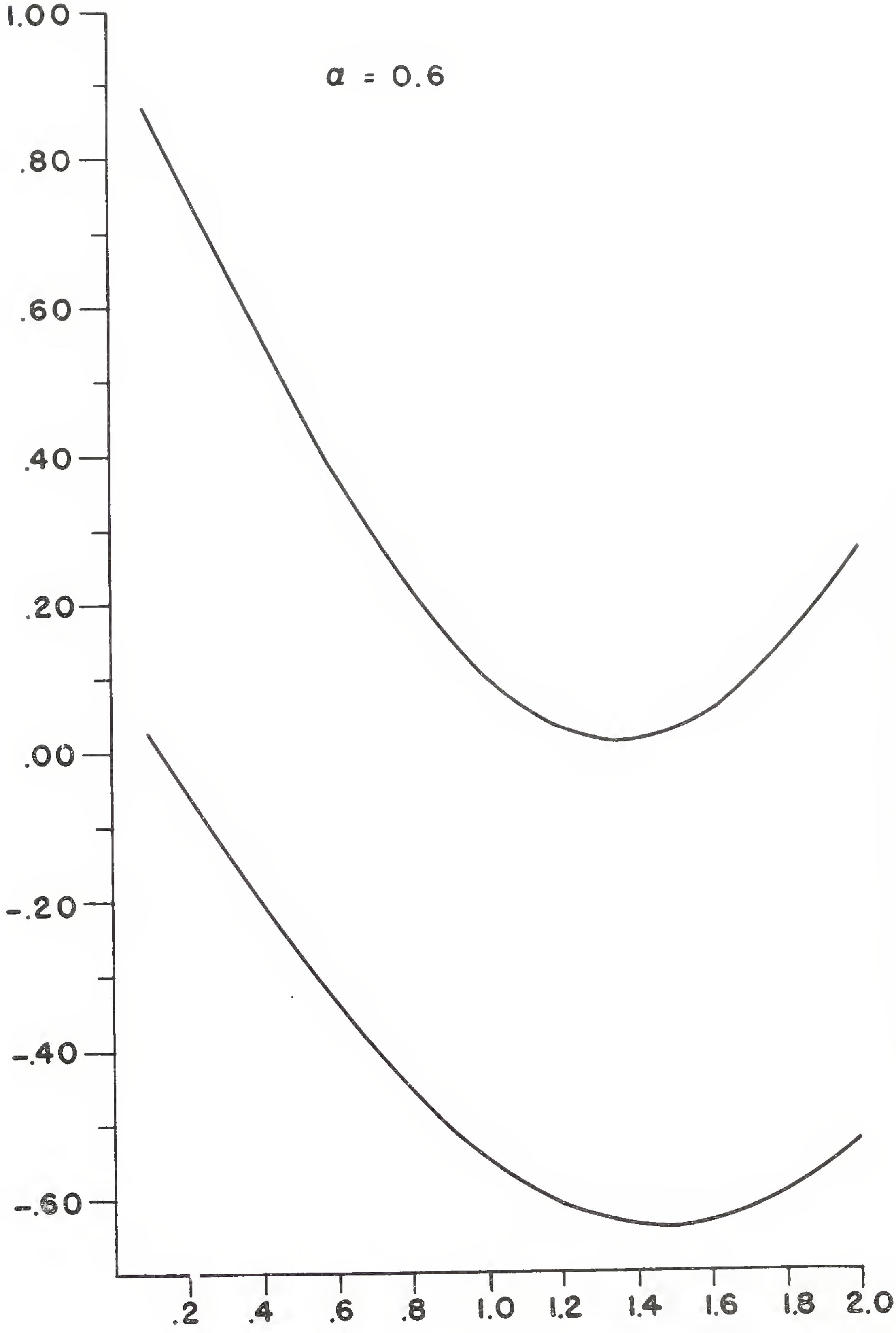


Fig. 3

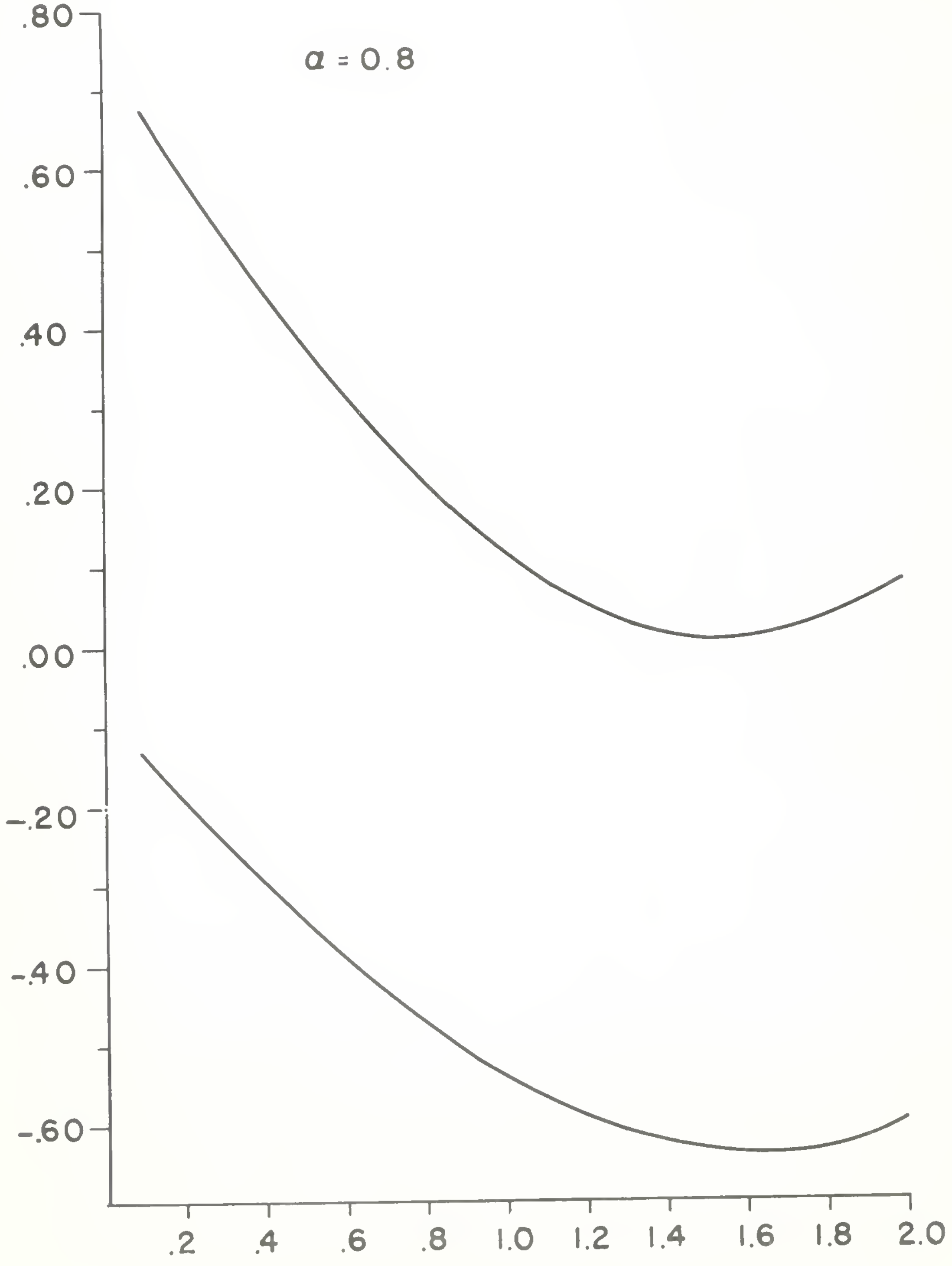


Fig. 4

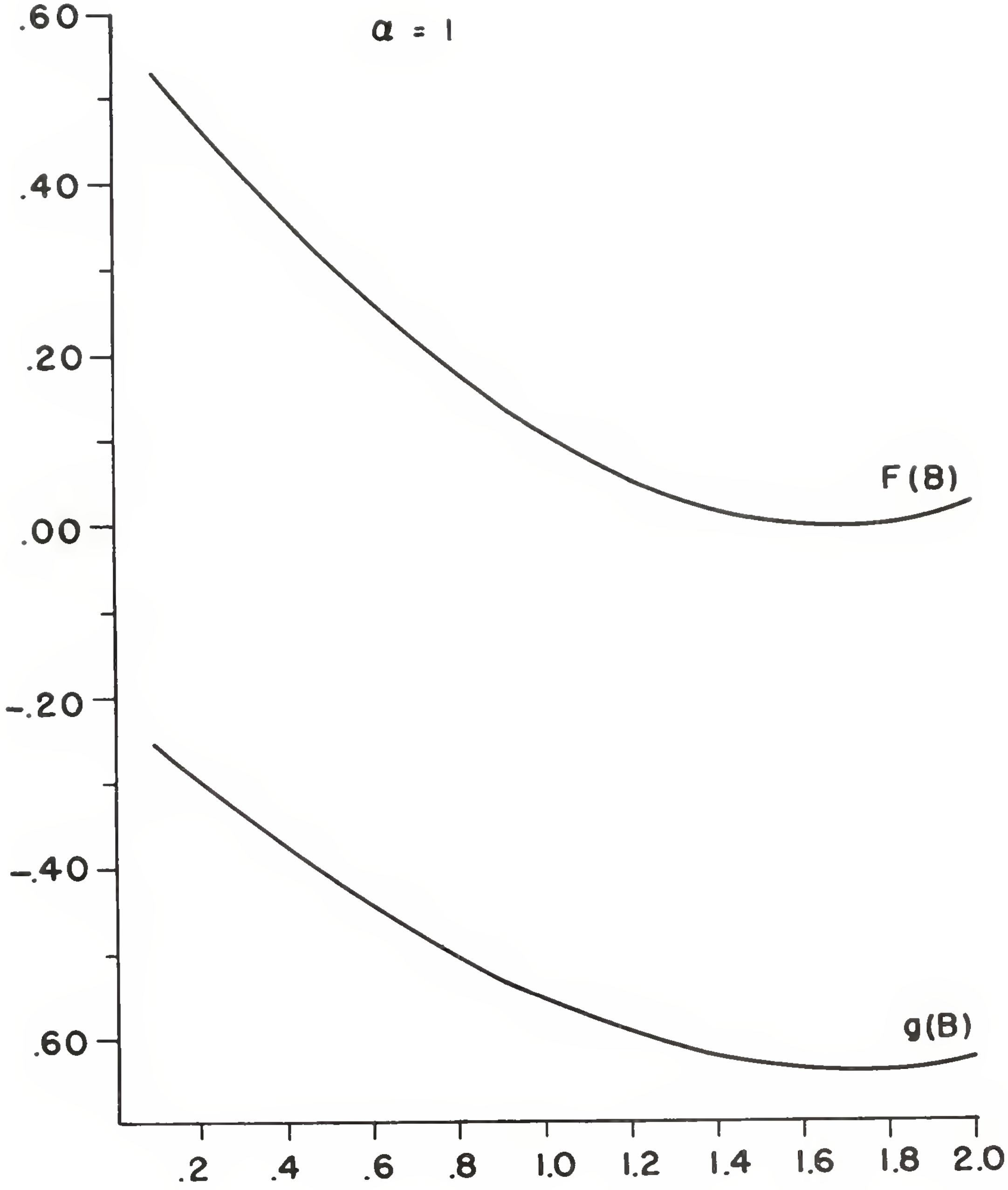


Fig. 5

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